Error Analysis of Class XI Students of SMA in Solving Matrix Problems in terms of Mathematical Ability

Muh. Hasbi 1*, Ester Lapalesa 2, Anggraini 3, Rahma Nasi 4

1,2,3,4 Tadulako University, Indonesia
E-mail korespondensi: esthevianalapalesa7794@gmail.com
DOI: 10.47435/jtmt.v4i02.1987

Abstract

This research aims to describe the types of errors made by students in solving matrix problems. The subjects in this research were three students of class XI MIPA SMA 5 Palu who were divided into three categories based on their level of mathematical ability, namely students with high, medium, and low mathematics ability. The selection of these three students was based on students report cards which were categorized into three types, namely high, medium and low, which were then selected through considerations, namely students who were able to take part in the entire of data collection activities carried out by researchers as well as considerations from mathematics subject at the school. This type of research is descriptive research with a qualitative approach. Data collection was carried out by administering written tests and interviews. The results of this research show that students of class XI MIPA SMA Negeri 5 Palu made mistakes in solving questions (1) high mathematics ability students made conceptual mistakes, namely students were wrong in writing the concept of matrix multiplication that had different orders. (2) students with moderate mathematics ability made procedural errors, namely students were wrong in carrying out the steps of solving the problem and students in solving the problem did not reach that students do not write the sign of mathematical operations that should be used. (3) students with low mathematics ability made conceptual errors, namely students did not solve the problems given, only rewrote the matrix members. Procedural errors, namely students are not careful in understanding the question instruction so that the steps for solving the problem are not appropriate and students do not solve the problem until the final stage. Technical errors, namely students are wrong in doing calculations so that the students answers do not match the alternative answers. This research is highly recommended for other researchers to explore further because the coverage of the matrix material is quite broad so researchers cannot discuss everything in this research and there are many more reviews that need to be analyzed through this material.

Keywords: Error Analysis, Matrix questions, Mathematical Abilities

1. Introduction

Mathematics is the science of everything, is the foundation for the development of modern technology, plays an important role in science, develops human thinking. Mathematics is very important for everyday life and to deal with the progress of science and technology, so mathematics should be taught to students from an early age. In line with opinion (Rizki, Yensy, & Rusdi, 2017) Mathematics is knowledge that exists in everyday life that can provide learning experiences that are very useful for the survival of life in the future. Therefore, mathematics has been taught at all levels, from elementary school to college. However, there are still many students who think that mathematics is a difficult
subject. Of the several materials in mathematics, matrices are one of the materials that students find difficult. This is in line with the pre-test conducted by (Farhadi & Munfarikha, 2020) He found that there was one student who achieved KKM scores from 33 students, this was because the majority of students could not perform matrix surgery.

Matrix is one of the materials taught at the upper secondary education level. This material can be used in everyday life such as facilitating economic problems that contain various variables. The study of matrix material is very broad including, types of matrices, matrix transpose, matrix similarity, and operations on matrices including addition, subtraction, multiplication of two matrices, matrix multiplication with scalars, determinants and matrix inverses. The breadth of study of matrix material is very much needed for students’ basic abilities, understanding concepts, problem solving and student skills in calculating, so that in the future students will not have difficulties in applying matrices in the field of mathematics learning and in everyday life. The reality in the field is that there are still many students who do not understand and master the concept of matrices, so students have difficulty in solving matrix problems which results in students making mistakes during the process of solving matrix problems. Error is a form of deviation from something that has been considered true or a form of deviation from something that has been agreed or determined previously (Rahmania & Rahmawati, 2016). Correspondingly (Ainin, Hartono, & Aripin, 2020) stated that data in the field shows that there are still many students who make various mistakes in solving matrix problems including student errors in determining matrix determinants, student errors in determining adjoin matrix as well as errors in performing calculations. The mistakes made by these students are feared to occur in several high schools including SMA Negeri 5 Palu.

Lipianto & Budiarto Mentioning that mistakes occur due to the low level of students' mathematical abilities that are less in-depth and less honed so that students make many mistakes (Lipianto & Budiarto, 2013). Then Lerner, posited that some common mistakes occur due to lack of understanding of mathematical operation symbols, lack of accuracy in calculations, incorrect use of processes or formulas and illegible writing. The causes of mistakes made by students are lack of understanding of the concepts of the material taught, lack of understanding of mathematical language or writing, students who are wrong in the use of mathematical formulas and inaccuracy of students in doing mathematical calculations (Baharuddin, Sukmawati, & Christy, 2021). Based on the opinions expressed above, it can be concluded that common mistakes that are often made by students are due to students' lack of understanding of the problems given, especially math problems, and students' lack of accuracy and skills in calculating. This is in line with previous research conducted by (Nuritasari, Hasanah, & Sholehoddin, 2017) which suggests that the location of students' errors in solving matrix problems is the error of understanding the problem, where students make mistakes in understanding what is known and what is asked on the problem, errors in solving the problem and errors in writing the final answer.

There are some common mistakes made by students in solving math problems according to Newman in (Harahap & Zahari, 2021) namely: 1) Reading error, 2) comprehension error, 3) Transformation error, 4) proses skill error, 5) Encoding error. In addition to Newman, Kastolan in (Firdaus, Amalia, & Zumeira, 2021) Also said that there are 3 types of mistakes that are often made by students, namely conceptual, procedural and technical errors. In this study, researchers used the type of error according to the kastolan stage because the type of error according to kastolan is easy to understand so that it can make it easier for researchers to use it.

Based on observations and interviews conducted by researchers on February 13, 2023, researchers found that many students make mistakes when completing matrices, including students making mistakes in multiplying two matrices because students do not understand and master the concept
that two matrices can be multiplied if the first-order matrix corresponds to the same number of matrices, columns such as the number of rows on the second matrix, as well as the process of multiplying the rows of the first matrix by the columns of the second matrix. In addition, many students have the habit of not being careful in seeing question commands and forgetting to write down mathematical symbols. The student’s error has variations or degrees, it occurs because students have varying levels of mathematical ability, namely students with high mathematical ability, students with moderate mathematical ability and students with low mathematical ability. According to (Devi, Usman, & Linawati, 2017) Mathematical ability is the ability or ability of students in relating mathematical concepts, representations and procedures, which are grouped into several different ability groups, namely high, medium and low abilities measured based on the results of mathematical ability tests. As stated by the Minister of Education and Culture in 2017 in (Nurma & Rahaju, 2021) suggests that students' mathematical abilities can be categorized into three, namely low mathematical ability, medium mathematical ability, and high mathematical ability. In line with research conducted by (Fitriani, Murdiana, & Rochaminah, 2021) that students with high mathematical ability have better accuracy than students with medium and low ability. So that the error rate made by high-ability students is small compared to medium and low-ability students. Students with moderate math skills are also better at solving math problems than students with low math skills. Therefore, researchers consider it necessary to conduct research related to the analysis of student errors in solving matrix problems in terms of their mathematical abilities. Through this analysis, the type and location of errors and an overview of mistakes made by students will be obtained, so that educators can provide appropriate solutions so that they can be corrected and not repeated and error information in solving math problems can be used to improve the effectiveness of mathematics learning.

2. Metode

This type of research is descriptive research with a qualitative approach, which is research that describes or provides an understanding of the mistakes made by students when solving matrix problems in terms of their mathematical abilities. This research was conducted at SMA Negeri 5 Palu. This research was conducted in the even semester of 2022/2023. The subjects selected in this study were students of class XI MIPA SMA Negeri 5 Palu consisting of three people with different levels of mathematical competence, namely students with high, medium, and low mathematical abilities. The selection of subjects is seen from the students’ report cards, as well as students who are able to follow the entire series of data collection by researchers and recommendations from teachers in the field of mathematics subjects. Data collection techniques in this study were carried out by providing researcher-led interviews and problem-solving exercises on each subject. This study used triangulation time to test the reliability of the data. A study requires data analysis so that research can be carried out properly and effectively, without any errors in the research process. Methods used in data analysis according to (Miles & Huberman, 1994) includes data compression, data presentation, inference, or verification

3. Results and Discussion

The error analysis of each subject is based on indicators according to Kastolan namely conceptual, procedural, and technical, presented in Table 1
Table 1. Kastolan Error Indicator

<table>
<thead>
<tr>
<th>No</th>
<th>Error Type</th>
<th>Error Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Conceptual Errors</td>
<td>a. Students are wrong in choosing, writing or applying formulas so that answers do not match</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Students don't do questions</td>
</tr>
<tr>
<td>2.</td>
<td>Procedural Errors</td>
<td>a. Students in solving questions are not in accordance with the question orders and question solving steps</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Students in solving problems do not arrive at the final stage or the simplest form</td>
</tr>
<tr>
<td>3.</td>
<td>Technical Errors</td>
<td>a. Students are wrong in counting</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Students are wrong in writing down the signs of mathematical operations that should be used</td>
</tr>
</tbody>
</table>

Source: (Firdaus et al., 2021)

The results obtained in this study are descriptions of the types of errors made by students with high mathematical ability (ST), students with medium mathematical ability (SS), and students with low mathematical ability (SR) in solving matrix problems based on Kastolan stage indicators. In this study, students were given written assignments related to the matrix material they had learned along with the questions given to students:

1) Given the following matrices $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ and $B = \text{determine (if any, as well as give a reason)}$:
   a. $A \times B$
   b. $B \times A$

2) Determine $B \times A$ from the following matrix:
   
   $$ A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} $$

3) Calculate the determinants of the following matrices:
   a. $\begin{pmatrix} 5 & 3 \\ 2 & 2 \end{pmatrix}$
   b. $\begin{pmatrix} 2 & -2 & 1 \\ 2 & 1 & -3 \\ 4 & 0 & -1 \end{pmatrix}$ (Hint: use the Sarrus method)

After the students did the problem, the researcher conducted interviews with all three research subjects. Then the researcher analyzed the results of student answers and interviews, then after analysis the researcher obtained the following results:
3.1 Error Analysis of High Mathematical Ability Students (ST)

The following are ST’s answers when solving the questions given.

Judging from the answer sheet of ST Subject, this shows that ST has made a mistake in question number 1 part a marked on the closing parentheses, which ST mistook the concept or condition of multiplication of two matrices. Excerpt of ST interview with Researcher (PE):

PE: Can you explain what information you know about it?
ST: First it is known that there are matrices A and B kak then the next is asked the multiplication of matrices A x B and B x A kak
PE: After knowing the command, do you think sister A x B can it be multiplied?
ST: You can’t because the number of columns of matrix A must be equal to the number of columns of matrix B

In line with the results of research conducted by (Afdila, Roza, & Maimunnah, 2018) also concluded the form of conceptual error, one of which is that students are not right in using the ST formula, writing that matrices A and B cannot be multiplied because the number of columns is different, the number of columns A is two, while the number of columns B is only one, the number of columns must be the same and then it can be multiplied. This is a conceptual error because the concept of matrix multiplication is that two matrices can be multiplied if the number of columns of the first matrix is equal to the number of rows of the second matrix.
Furthermore, question number 2 shows that ST is able to solve the problem according to the steps requested, ST understands well the concept of matrix multiplication of the same order and ST is also able to solve the problem to the final stage without any errors in doing calculations.

From the explanation above, we can conclude that the mistakes made by students with high mathematical ability from the three questions given are conceptual errors located in number 1. This is in line with the results of research conducted by (Nuritasari et al., 2017) that the kind of mistake often made by students is a conceptual mistake. The factor causing student error is due to students who do not understand their mathematical concepts.
3.2 Error Analysis of Students with Moderate Mathematical Ability (SS)

The following will be presented SS answers when solving the questions given.

![SS answer to question number 1](image)

**Figure 4. SS answer to question number 1**

Judging from the SS answer to question number 1, SS answered the question according to the concept of matrix multiplication of different orders. SS answers on question number 1 part a that A x B cannot be done because the number of columns of matrix A must be equal to the number of rows of matrix B, and this is in accordance with the concept of matrix multiplication, it can be concluded that SS did not make a mistake in question 1 part a, then in question part b SS answered that B x A can be multiplied because the number of columns B is equal to the number of rows of matrix A, and the SS also did the calculations for the question and the answers found were also in accordance with the alternative answers, so the SS also did not make a mistake on question number 1 part B.

![SS answer to question number 2](image)

**Figure 5. SS answer to question number 2**

Judging from SS's answer above that SS had made a mistake marked in the closing bracket above, it appears that SS was wrong in solving the problem where SS solved the problem not to the final stage or to the simplest form. SS was also wrong in performing the steps of multiplication of a matrix of order 3 x 3. In the first step SS was correct in multiplying the first row of matrix A and the first column of matrix B, but in the next step SS had made a mistake so we can conclude that SS had made a procedural error.
Judging from SS's answer, we can also know that SS had made a mistake, namely a technical error because SS miswrote the sign of the mathematical operation that should be used. In the above problem there should be a less operating sign (-) so that it can be calculated 10-6 and the determinant result can be obtained, which is 4. However, the SS did not do so, so it can be concluded that the SS had made a technical mistake.

From the above explanation, we can conclude that the mistakes made by the SS were procedural and technical errors. This is in line with research conducted by (Maryani & Chotimah, 2020) That is, there are student mistakes made when solving matrix problems, namely 1) students do not use matrix symbols or signs of mathematical operations, 2) students do not do the problem systematically, 3) and students are wrong in calculating the final results.

3.3 Error Analysis of Students with Low Mathematical Ability (SR)

The following will be presented SR's answers when solving the problem

Figure 7. SR answer to question number 1

Makna dari Gambar 7 adalah SR atau siswa berkemampuan rendah tidak mengerjakan soal yang diberikan oleh peneliti. SR hanya menuliskan kembali soal yang berikan maka dapat kita simpulkan bahwa SR telah melakukan kesalahan konseptual sesuai dengan indikator yang dipaparkan oleh peneliti diatas pada point 1 bagian b yakni siswa tidak mengerjakan soal.

The meaning of Figure 7 is that SR or low-ability students do not do the problems given by the researcher. SR only writes back the questions given, so we can conclude that SR has made conceptual errors in accordance with the indicators described by the researcher above in point 1 part b, namely students do not do the questions.
Figure 8. SR answer to question number 2

From figure 8 above, the low student (SR) writes the answer, matrix A multiplied by matrix B, then the results are summed, this is correct in the process of solving it, but SR incorrectly writes the order of the matrix in accordance with the question order, indicated by the closing brackets above. The question command given by the researcher is B X A but SR writes A X B so this is an SR error, besides that SR also does not solve the problem until the final stage, so on this data the researcher concludes that SR has made a procedural error.

Figure 9. SR answer to question number 3

From figure 9 above, this SR answer found an error in part b of the question. In part a, SR's answer is correct, but in part b, there is a part of the problem solving process that is not written by SR so that SR made a mistake in calculating the final answer, so we can conclude that SR has also made a technical error. From the explanation above, we can conclude that the mistakes made by SR are procedural and technical errors. This is in line with research conducted by (Ainin et al., 2020) which states that students' errors in solving problems are because students do not understand the concept of the given problem and errors in calculations. Research results (Maryani and Chotimah, 2020) also stated that students did not carry out the calculation process as in the subject of this study, namely the SR subject on question number 3, students did not understand the problem, and students did not complete the problem until the final stage.
4. Conclusion

Based on the results of research and discussion, it was concluded that, the description of errors made by students with high, medium and low mathematical abilities in solving matrix problems based on Kastolan’s error indicators was (1) students with high mathematical abilities made concept errors, namely students were wrong in writing down the concept of matrix multiplication that had different orders. (2) Students with mathematical ability are making procedural errors, namely students are wrong in completing the problem solving steps and students in solving the problem do not reach the final stage. The technical error is that students do not write down the signs of mathematical operations that should be used. (3) Students with low mathematical ability make conceptual mistakes, that is, students do not solve the given problems, only rewrite the matrix members. Procedural errors are students not careful in understanding the question commands so that the steps for solving the questions are not appropriate and students do not complete the questions until the final stage. Technical errors are students who make wrong calculations so that students’ answers do not match the alternative answers.

Based on the results of the research and discussion presented, the advice that can be given in this study is that teachers and prospective teachers can provide more practice to students so that the mistakes made can be reduced and the level of mathematical ability can also increase. Furthermore, prospective researchers who will explore further related to this research pay more attention to the parts of the matrix material that need to be researched such as the matrix inverse and other matrix parts so that the reader's understanding of the material is even wider.

Bibliography


contextual teaching learning di SMA Negeri 1 Portibi. 8(2), 562–575.


